22. ANS:

By definition, $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

$f(x) = \tan\left(\dfrac{1}{x}\right)$

$$f'(x) = \lim_{h \to 0} \frac{\tan\left(\frac{1}{x+h}\right) - \tan\left(\frac{1}{x}\right)}{h}$$

$$= \lim_{h \to 0} \frac{\sin\left(\frac{1}{x+h}\right)\cos\left(\frac{1}{x}\right) - \cos\left(\frac{1}{x+h}\right)\sin\left(\frac{1}{x}\right)}{h\cos\left(\frac{1}{x}\right)\cos\left(\frac{1}{x+h}\right)}$$

$$= \lim_{h \to 0} \frac{\sin\left[\frac{1}{x+h} - \frac{1}{x}\right]}{h\cos\left(\frac{1}{x}\right)\cos\left(\frac{1}{x+h}\right)}$$

$$= \lim_{h \to 0} \frac{-\sin\left(\frac{h}{x(x+h)}\right)}{h\cos\left(\frac{1}{x}\right)\cos\left(\frac{1}{x+h}\right)}$$

$$= \lim_{h \to 0} \frac{-\sin\left(\frac{h}{x(x+h)}\right)}{\frac{h}{x(x+h)}} \times \frac{}{x(x+h)} \times \frac{}{\cos\left(\frac{1}{x}\right)\cos\left(\frac{1}{x+h}\right)}$$

$$= -1 \times \frac{}{x^2} \times \frac{1}{\cos^2\left(\frac{1}{x}\right)}$$

$$\frac{-1}{x^2}\sec^2\left(\frac{1}{x}\right)$$

REF: Application OBJ: Chapter 4 Problems LOC: DAV.01, DA1.04
TOP: Using the Derivative to Analyze Polynomial Function Models